Structuralism - Its prominent past, sad present, and bright future WORKSHOP

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Integrative Methods of Inquiry in Education: Symmetry

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At the beginning was…

Where to start?

Should we start from “structure”? What is it?

(The audience provides a clear and precise definition of the concept of a structure -:)

Should we start from “symmetry”? What is it?

(The audience provides a clear and precise definition of the concept of symmetry -:)

So, maybe we should start from a mirror?

(Finally everyone knows what we are talking about!)

Or at least everyone believes it is obvious.

Magic Mirror that Tells the Truth

After all almost all children in the world know about the Magic Mirror:

1)Lustereczko powiedz przecie kto jest najpiękniejszy w świecie? Ty, Krȯlowo.

2)「鏡よ鏡、世界で一番美しいのはだあれ？」
「それは王妃様です」

3) Mirror, mirror in my hand who is fairest in the land? You, Your Majesty.

But finally this can happen:

Mirror, mirror in my hand who is fairest in the land?

I am sorry Your Majesty, Snow White.

WHAMM!!! (Symmetry is broken…)

Magic Mirror that Tells the Lie

There was always universal fascination with mirrors in diverse cultures.

Just an example: *shinkyō* (sacred mirror) in Shintō rituals serving as *mishōtai* (the divine true body), typically of Amaterasu Ōmikami.

But, does the mirror actually tell the truth?

Do we see in the mirror our face the same way others see us? Of course not! Our left side is exchanged with the right side. **So, why don't not we see ourselves upside down?**

Magic? Mystery?

Mirror Symmetry Born 1794?

Real mystery is in the fact that what we call "mirror symmetry" was defined in a clear way so late!

Giora Hon and Bernard R. Goldstein "From Summetria to Symmetry: The making of a revolutionary scientific concept" Archimedes: Springer, 2008.

Hon & Goldstein demonstrate through literature review that the first occurrence of the modern meaning of mirror symmetry in a very specialized context of geometry in **1794** in the work of **Adrien-Marie Legendre** "*Eléments de géométrie*" :

"Two equal solid angles which are formed (by the same plane angles) but in reverse order will be called angles equal by symmetry, or simply symmetrical angles."

Mirror Symmetry Born-Again 1872

Hon & Goldstein trace the **first clear definition** after failed attempts of Leonhard Euler and Immanuel Kant to **Ernst Mach's lecture "On Symmetry"** published in **1872**:

"If […] we can divide an object by a plane into two halves so that each half, as seen in the reflecting plane of division, is a mirror image of the other half, such an object is termed symmetrical, and the plane of division is called the plane of symmetry."

The way from "symmetric" understood as "proportional", "commensurable", "harmonious" to modern meaning ends.

But This Is Just the Beginning

Mirror Symmetry has been associated with invariance with respect to transformation, in this case mirror reflection of the points of space.

There is a natural question about invariance with respect to other transformations of space.

We will restrict our understanding of a geometric space to the plane to simplify our considerations.

Thus, the mirror reflection in this case is in a line and a simple example of a symmetric object is in this case a square which can be divided by the reflecting lines passing through its center and parallel to the sides or by the lines including its diagonals.

Erlangen Program of Felix Klein 1872

The explosion of the studies of symmetry came with the idea of considering any kind of geometry (by 1872 there were many) as a study of invariants of geometric transformations.

Klein, F. C. (1872/2008). A Comparative Review of Recent Researches in Geometry (*Vergleichende Betrachtungen über neuere geometrische Forschungen*). Haskell, M. W. (Transl.) arXiv:0807.3161v1

To understand this idea better we have to introduce an overview of basic concepts of geometry and algebra.

Let's start from geometry.

Euclidean Geometry on a Plane

Five Principles (historically not accurate):

1) **Through every two points there is exactly one line passing.**
2) **Every segment can be extended indefinitely to a line**
3) **Every two right angles are congruent.**
4) **For every point and every segment there is exactly one circle which has this point as a center and the radius equal to this segment**
5) **For every line and point that does not belong to it there is exactly one line parallel (not having common points) to the given one which is passing through the given point.**

Non-Euclidean Geometries on a Plane

In the 19th century it was discovered that the fifth postulate

"For every line and point that does not belong to it there is exactly one line parallel to the given one which is passing through given point"

can be replaced by either one with many parallel lines passing or by one in which no line is passing. This was a first type of diversification of geometries. Another variation either eliminated the concept of right angle (affine geometry) or imposed the condition that every two lines have points in common (projective geometry incorporating the "vanishing points" of the perspective).

Groups of Transformations

Another development stimulating the idea of Erlangen Program was a new concept of a group. It came out of the consideration of composition of transformations (mappings that establish one-to-one correspondence between all elements of the transformed set) which itself is a transformation.

Thus as in the case of numbers for which we have operations (addition or multiplication) which produce a number, we can think about the operation of composition on transformations which produces a transformation.

It turns out that both the operations on numbers and the operation on transformations can be generalized.

Concept of a Group in General (1)

Let's consider a set G of objects (numbers, transformations, …) for which we have a binary operation $*$, i.e. for all x, y in G, z = x*y belongs to G.

We do not require that x*y = y*x (as it is for addition or multiplication of numbers), but only that operation is associative: (x*y)*z = x*(y*z).

Now, if an element e of G is such that for every x in G we have x*e = e*x = x, we call e a neutral element or identity. Of course for composition of transformations the transformation which assigns to each element itself is neutral.

It is easy to show that no matter what is the set G and what is the operation $*$, there can be at most one neutral element.

Concept of a Group in General (2)

If an element x’ of G is such that **for every x** in G we have **x∗x’ = x’∗x = e**, then we call **x’ an inverse of x**. For the composition of transformations the inverse transformation is “undoing” the original transformation. It is easy to identify the inverse x’ of any number x with respect to addition. In the case of multiplication there is only one number for which there is no inverse.

It is easy to show that no matter what is the set G and what is the operation ∗, **the inverse element is unique.**

DEF. The set G with an associative operation ∗, which has a neutral element and inverse for each element is called a GROUP.

Example: The set of all transformations of any set S is a group with respect to composition.

Concept of a Group in General (3)

We need two more concepts. Let <G,∗> be a group.

DEF. Subset H of G is a subgroup of <G,∗> , if for all x, y in H, z = x∗y belongs to H and x’ belongs to H. This means that H is closed with respect to the operation ∗ and with respect to taking inverse.

Finally we consider mapping (function) φ: G → H between two groups <G,∗> and <H,∘> .

DEF. If function φ: G → H satisfies the conditions:

φ(x∗y) = φ(x) ∘ φ(y) and φ(x’) = φ(x)’ , i.e. function φ preserves the operation and taking inverse, we call the function φ a group homomorphism. If the function φ is one-to-one and onto it is called an isomorphism.

Two groups are isomorphic (from mathematical point of view the same), if there exists an isomorphism between them.

Famous Example of a Group

In Structuralism one tiny group, Klein Group, played an exceptional role. It is defined on a small set G={e,a,b,c}.

Its operation is described by the following Cayley table:

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Alternatively we can describe it by the rule that: $a^2=b^2=(ab)^2=e$ and $a\neq b \neq ab$, $ab=ba$ where the operation is indicated by the juxtaposition.

The subgroups are: {e}, {a,e}, {b,e}, {c,e}, {a,b,c.e}.

This group is a symmetry group for a rectangle and for a rhombus which are not squares.

Back to Erlangen Program of Felix Klein

Klein recognized that different types of geometry are characterized by different groups of transformations which preserve their geometric structure. For instance Euclidean geometry is distinguished by the group of transformations which preserve distance.

Moreover the mutual relationships between geometries can be investigated through the analysis of the mutual relationship between their symmetry groups.

Finally, properties of specific geometric objects can be studied through the analysis of subgroups of transformations for which they are invariants.

In the last case we can see that rectangles and rhombuses have the same symmetry type described by Klein's Group.

Important Lesson from Klein

Really important in Erlangen Program, but not always remembered in the attempts to emulate it in different contexts is:

1) Symmetry is always in the context of the type of transformations. In Klein's Program it is the distinction of the type of geometry determined by the symmetry of entire space.
2) Symmetry of particular objects are relative to the symmetry of space. It is determined by the subgroup of the total group.
3) Important information is not only in invariant subsets (sets whose points are "moving", while entire set remains unchanged), but also in stabilizers (sets of points which are not "moving" at all)

Magic in Mirror Symmetry

Mirror symmetry became just one of many types of symmetry, but its magic remains.

It is interesting, if not mysterious that all other transformations within Euclidean group (group of all transformations preserving Euclidean distance, i.e. group of isometries) can be constructed from mirror reflections.

For instance on the plane: every rotation around given point is a double mirror reflection with respect to lines intersecting at this point.

We Are Done With Mirrors and Symmetries

Now what about structures and structuralism?

In mathematics and physical sciences situation was relatively clear. The concepts of algebraic structures (such as groups), topological structures (related to the intuitive concept of continuous transformations), etc. became everyday tools.

Outside of science the concept of a structure started to be used in the end of 19th century.

However, what does it mean “structure” in general still was mystery.

Beginning of Structuralism

Often associated with the work of **Ferdinand de Saussure** on linguistics (more specifically his lectures 1907-1911 posthumously published by his disciples in 1916).

The turning point - The distinction of two fundamental types of inquiry: **Diachronic and Synchronic** and shift from the former to the latter in the humanities and social sciences.

Thus far the dominating approach was to trace historical evolution of the subject of study and to seek their origin.

Methodology of de Saussure focused on the synchronic **structure** (of language). His central tool was the **concept of difference**.

Nebulous Concept of Structure

Increasing interest in the concept of a structure had a broad historical background. It can be associated with the parallel but very different approach to inquiry outside of the orthodox scientific methodology.

Science developed using as primary intellectual tools quantitative description and reductionist methodology.

The latter was blamed for the shortcomings of the scientific view of the world, in particular for the insufficient progress in the study of mind, life and complex objects of study.

Holistic Methodology (1)

Although the term holism was introduced by **Jan Christiaan Smuts** as **"the tendency in nature to form wholes that are greater than the sum of the parts through creative evolution"** ("Holism and Evolution" 1926) less than a hundred years ago, we can identify holistic tendencies in philosophical reflection through all ages and across all civilizations.

Holistic Methodology (2)

In the more general perspective holism can be understood as an epistemological or ontological position regarding the one-many relationship in dualistic opposition to reductionism.

It is the negative answer to the question whether the wholes can be reduced to their constituents (in explanation, or in reality).

General System Theory

General System Theory (GST) introduced by **Ludwig Von Bertalanffy** (1950) and continued by Kenneth E. Boulding, William Ross Ashby, Anatol Rappaport generated great hope for a new chapter in the philosophy and methodology of science.

It is no surprise that Von Bertalanffy was a biologist working on mathematical models of the growth of organisms. Piaget considered him a pioneer of structuralism in biology.

Von Bertalanffy, L. (1950) An Outline of General System Theory. *British Journal for the Philosophy of Science*, 1, 134-165.

What is a System?

The main obstacle in making General System Theory anything more than just fleeting fashion was the lack of sound conceptual foundations. Even the definition of the system remained nebulous.

Dictionary: A system is a regularly interacting or interdependent group of items forming a unified whole.

Merriam-Webster. Springfield, MA, USA.

Frequent association System – Structure, such as "a system is a set of components equipped with relatively stable structure"

Structuralism Triumphant (1)

The works of **Jean Piaget** in developmental psychology and of **Claude Levi-Strauss** in cultural anthropology in 1960's brought a lot of attention to the emerging philosophical direction promising **reconnection of the drifting apart Two Cultures of C.P. Snow.**

Triumph of Structuralism at that time definitely was influenced by the publication of the book "Symmetry" written by a highly recognized mathematician Hermann Weyl in which he showed how Erlangen Program can be extended to the study of culture and humanities.

Structuralism Triumphant (2)

Interesting and promising cooperation:

Evert Willem Beth highly recognized Dutch logician was very critical about Piaget's enthusiasm, but accepted Piaget's invitation to work together. This resulted in:

Beth, E.W. & Piaget, J. (1974) Mathematical Epistemology and Psychology. Springer: Berlin.

What Happened? Post-Structuralism?

Accusations:

Ahistorysm, Dryness, Garbage-In-Garbage-Out

Real Sins:

Levi-Strauss was using Klein Program as a metaphor rather than methodology:

Mauro W. Barbosa de Almeida (1990) Symmetry and Entropy: Mathematical Metaphors in the Work of Levi-Strauss. Current Anthropology, 31 (4), 367-385.

Bad Example

Alison Assiter (1984) Althusser and Structuralism. British J. of Sociology, 35(2), 272-296.

Four ideas of structuralism:

- Structure determines the position of each element of the whole,
- Every system has a structure,
- Structural laws deal with co-existence rather than change,
- Structures are the ‘real things’ that lie beneath the surface or the appearance of meaning.

WHAT !!!???

Reminder:Important Lesson from Klein

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THE BRIGHT FUTURE?

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